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LETTER TO THE EDITOR

Critical dynamics of the alternating bond kinetic Ising model

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Abstract. The critical dynamics of the alternating bond kinetic Ising model on a chain is examined. It is shown that previous non-universal results for the dynamical critical exponent z for single spin flip dynamics should be reinterpreted. The critical slowing down is due to two distinct contributions which, when separated, indicate that the value of z is universal. The exponent remains universal even when two spin flip dynamics are considered.

There has recently been a great deal of interest in violations of standard dynamic scaling theory [1] in the relaxational dynamics of Ising spin models of systems which have a critical point at zero temperature. These systems include critical percolation clusters [2], regular fractals [3, 4] and inhomogeneous chains [5, 6]. The simplest dynamics for the Ising model is that proposed by Glauber [7] in which transitions between states occur due to the flipping of single spins. Glauber solved this model exactly for the case of a homogeneous chain of Ising spins interacting with nearest neighbour interactions and it is one of the few exactly solvable kinetic models. Both the time-dependent magnetization and pair correlation function can be obtained and the model exhibits non-trivial critical slowing down near $T_c = 0$. The conventional scaling theory describes the dependence of the relaxation time τ on the divergent correlation length ξ as follows

$$\tau \sim \xi^z \tag{1}$$

where z is the dynamical critical exponent. The Glauber model predicts the value $z = 2$. Generalizations of this model have been used to study systems with a conserved order parameter [8] and various universality classes for z have been identified using the renormalization group [9] in dimension $d = 4 - \varepsilon$.

The master equation of the kinetic Ising model is

$$\frac{\mathrm{d}}{\mathrm{d}t}P(\{S\},t) = -\Gamma\sum_i (1 - p_i)W_i(S_i)P(\{S\},t) \tag{2}$$

where $P(\{S\},t)$ is the time-dependent spin probability distribution, p_i is the spin flip operator, Γ^{-1} is a bare time scale for an isolated spin and $W_i(S_i)$ is the probability

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transition rate of the ith spin from a state S_i to a state $-S_i$. The transition probability rate satisfies the detailed balance relation

$$(1 - p_i)W_i(S_i)P_e(\{S\}) \tag{3}$$

where $P_e(\{S\})$ is the equilibrium distribution. This condition ensures the ergodicity of the system but does not uniquely determine the form of $W_i(S_i)$. Several authors [10–12] have studied the question of whether the exponent z depends on the choice of the single spin probability W_i. In particular, Deker and Haake [10] show that for a special choice of W_i the value of z for the homogeneous chain can be changed from two to four. However, for this special case there are an infinite number of states [13] which are stable against single spin flips. They also conclude that the consideration of multiple-spin flip rates is not necessary to find z.

Droz *et al* [5] reported that for a given choice of the single spin rate, the exponent z is non-universal in a system with inhomogeneous couplings. They found that the alternating bond Glauber–Ising model does not belong to the universality class of the uniform chain with the dynamic exponent $z = 2$ but has a non-universal value $z = 1 + (J_2/J_1) \geqslant 2$, where J_1 and J_2 are the alternating bond strengths with $J_2 \geqslant J_1$. As described below, a more detailed study of the dynamics of this model shows that this asymptotic behaviour is a result of two different contributions. One is due to long ranged fluctuations near the critical point and the other is due to short ranged phenomena.

The one spin flip model is a straightforward generalization of Glauber's choice of the single spin flip rate W_i to this system [5, 6]

$$W_i = \tfrac{1}{2}\left[1 - a_i^+ S_{i-1}S_i - a_i^- S_iS_{i+1}\right] \tag{4}$$

where

$$a_i^\pm = \tfrac{1}{2}\left[\tanh(J_i + J_{i+1}) \pm \tanh(J_i - J_{i+1})\right] \tag{5}$$

with $J_{2i} = J_1$, $J_{2i+1} = J_2$ and $J_2 > J_1$. The average value of each spin is given by the solution of

$$\frac{\mathrm{d}}{\mathrm{d}t}q_i = \frac{\mathrm{d}}{\mathrm{d}t}\langle S_i\rangle = -2\Gamma\langle S_iW_i(S_i)\rangle \tag{6}$$

where the average $\langle\ \rangle$ is taken with respect to $P(\{S\}, t)$. A Fourier and Laplace analysis of (6) with respect to momentum k and the frequency ω respectively leads to the following dispersion relation

$$\omega/\Gamma = 1 \pm \left[\tanh^2(J_2 + J_1) + (\tanh^2(J_2 - J_1) - \tanh^2(J_2 + J_1))\sin^2 k\right]^{1/2}. \tag{7}$$

This dispersion relation describes two bands of characteristic frequencies. In the limit of low T the width of both bands is proportional to $\mathrm{e}^{-2(J_2-J_1)}$. Each mode in the lower band corresponds to a single spin flip metastable state with a relaxation time that diverges at $T = 0$. However, this divergence has nothing to do with critical phenomena and is present in any chain with inhomogeneous couplings. In the alternating chain, there are strong bonds(J_2) and weak bonds(J_1). The spins that are coupled by strong bonds form blocks which have their spins aligned. At low T,

the lowest energy excitations correspond to these blocks flipping as a unit because in this case only the weak bonds connecting the blocks are broken. If the characteristic frequencies are measured in units of the band width, then the internal structure of the dispersion relation is the same as in the homogeneous chain.

In the limit of small k, the slow mode behaves as

$$\omega_c \sim \Gamma e^{-2(J_2-J_1)}(\xi^{-2} + k^2)$$

where the correlation length $\xi = e^{2J_1}$ only depends on the weakest bond. This can be rewritten in the form

$$\omega_{II} \sim \Gamma e^{-2(J_2-J_1)} k^2 (1 + (k\xi)^{-2})$$

for $k\xi \to \infty$ and as

$$\omega_{III} \sim \Gamma e^{-2(J_2-J_1)} \xi^{-2} (1 + (k\xi)^2)$$

for $k\xi \to 0$. To identify z from these asymptotic expressions they should be compared to the functional form of the characteristic time scale, $\omega_c(k) \equiv \tau_c^{-1} = k^z \Omega(k\xi)$ as predicted by the dynamic scaling hypothesis [1]

$$\begin{aligned} &\text{region II}: \ k\xi \gg 1, \ T \approx T_c \qquad \omega_c(k) = k^z \Omega(k\xi), \ \Omega(k\xi) \to \text{constant} \\ &\text{region III}: \ k\xi \ll 1, \ T - T_c > 0 \qquad \omega_c(k) = \xi^{-z} f(k\xi), \ f(k\xi) \to \text{constant}. \end{aligned} \tag{8}$$

The comparison shows that the model has two time scales. The first one is the bare time scale $\tau_0 \sim \Gamma^{-1} e^{2(J_2-J_1)}$, which is due to short ranged effects and characterizes the width of the two bands of the spectrum. The second time scale, $\sim \xi^2$ is due to the long ranged fluctuations. This time scale is responsible for the detailed structure of the dispersion relation. Therefore, when the bare time scale due to short ranged effects is separated from the time scale due to long ranged fluctuations, the dynamic exponent z is clearly identified and has the same value as in the homogeneous chain. The same conclusion can be reached using exact renormalization group transformations and these results will be reported separately [14].

Any inhomogeneous Ising chain has an exponentially large number of states at zero temperature [15] which are metastable against single spin flips. Hence the important excitations in the model do not correspond to single spin flips. There are an infinite number of divergent relaxation times in the single spin flip dynamics with each one corresponding to a metastable state.

An alternative approach to describe the dynamics of the one-dimensional chain could include a mechanism for the metastable states to relax to equilibrium. In the case of the alternating bond chain, such a mechanism would correspond to the simultaneous flip of the two spins coupled by a strong bond. The dynamic scaling hypothesis [1] predicts that kinetic models which are different only on a short range scale should belong to the same universality class and hence we would expect that $z = 2$. In order to allow the metastable states to relax, the following transition rate probability can be added

$$\widetilde{W}_i(S_{2i}S_{2i+1}) = \tfrac{1}{4}\Gamma'(1 + S_{2i}S_{2i+1})\left[1 - \tfrac{1}{2}S_{2i}(S_{2i-1} + S_{2i+2})\tanh(2J_1)\right]. \tag{9}$$

This transition rate allows the two aligned spins which form a block to flip together. It satisfies the detailed balance condition, $(1-p_{2j,2j+1})\widetilde{W}_j(S_{2j}S_{2j+1})P_e(\{S\})=0$ where $p_{2j,2j+1}$ is a two-spin flip operator. The master equation becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}P(\{S\},t)=-\left[\sum_i(1-p_i)W_i(S_i)+\sum_j(1-p_{2j,2j+1})\widetilde{W}_j(S_{2j}S_{2j+1})\right]P(\{S\},t). \tag{10}$$

The average value of the spin is now determined by

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle q_i\rangle=-2[\langle S_iW_i(S_i)\rangle+\langle S_i\widetilde{W}_j(S_{2j}S_{2j+1})\rangle] \tag{11}$$

where $j=\mathrm{integer}(i/2)$ labels the blocks. The time-dependent magnetization of the jth block, $m_j\equiv\sum_{i=2j}^{2j+1}q_i$, obeys the kinetic equation

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}m_j=&-\Gamma\left[q_{2j}-a_1^+q_{2j-1}-a_1^-q_{2j+1}+q_{2j+1}-a_2^+q_{2j}-a_2^-q_{2j+2}\right]\\&-\tfrac{1}{2}\left\langle\Gamma'(S_{2j}+S_{2j+1})\left[1-\tfrac{1}{2}(S_{2j}+S_{2j+1})(S_{2j-1}+S_{2j+2})\tanh(2J_1)\right]\right\rangle.\end{aligned} \tag{12}$$

The average in (10) involves the term

$$\tfrac{1}{2}\langle(S_{2j}+S_{2j+1})^2(S_{2j-1}+S_{2j+2})\rangle=2\sum{}'\sum_{(S_{2j}S_{2j+1})=+1}(S_{2j-1}+S_{2j+2})P(\{S\},t) \tag{13}$$

where $\sum'$ is the trace over spins that are not in the jth block. This term can be rewritten as

$$\langle(S_{2j-1}+S_{2j+2})\rangle-\sum{}'\sum_{(S_{2j}S_{2j+1})=-1}(S_{2j-1}+S_{2j+2})P(\{S\},t). \tag{14}$$

The relative magnitude of the two terms in (12), in the linear response regime, can be evaluated using $P_e(\{S\})$ instead of $P\{S\},t)$. By using standard transfer matrix methods we obtain

$$\frac{\sum'\sum_{(S_{2j}S_{2j+1})=-1}(S_{2j-1}+S_{2j+2})P(\{S\},t)}{\sum'\sum_{(S_{2j}S_{2j+1})=+1}(S_{2j-1}+S_{2j+2})P(\{S\},t)}\sim\mathrm{e}^{-2(J_1+J_2)}. \tag{15}$$

Thus, the last term of (12) can be neglected, and (11) becomes $\approx q_{2j-1}+q_{2j+2}$ The total magnetization $M(t)$ relaxes according to

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}M(t)&=-M(t)\left[\Gamma\{1-(a_1+a_2)\}+\Gamma'\{1-\tanh(2J_1)\}\right]\\&=-M(t)\left[\Gamma\{1-\tanh(J_2+J_1)\}+\Gamma'(1-\tanh(2J_1)\right].\end{aligned} \tag{16}$$

Two time scales now characterize the relaxation of the magnetization. The leading time scale is

$$\tau' \sim [1 - \tanh(2J_1)]^{-1} \sim \xi^2 . \tag{17}$$

The inclusion of the multiple spin flips introduces a relaxation mechanism for the single spin flip metastable states and this mechanism dominates the relaxation to equilibrium near T_c. Hence the critical slowing down has the same asymptotic singularity as in the uniform Ising chain.

The probability transition rate in (7) is not symmetrical in the sense that only the blocks composed of strongly interacting spins are allowed to flip. We should also add the transition rate

$$\widetilde{\widetilde{W}}_i(S_{2i-1}S_{2i}) = \tfrac{1}{4}\Gamma''(1 + S_{2i-1}S_{2i})\left[1 - \tfrac{1}{2}S_{2i-1}(S_{2i-2} + S_{2i+1})\tanh(2J_2)\right] \tag{18}$$

which contributes to (14) a term $\Gamma''\{1 - \tanh(2J_2)\}$. This term is smaller than the other two terms and thus can be neglected.

The results obtained here can be generalized to other inhomogeneous chains. One case is a translational invariant chain with a unit cell composed of many different interactions. This system also has an exponentially large number of states which are metastable against single spin flips. If one allows the blocks of spins which are between the weakest bonds to flip together, the conventional result for z is also found. Another case is the completely random chain where the weak bonds are distributed in an irregular way. If we allow multi-spin flips, then the dynamics as well as the statics are controlled only by the weakest bonds and the conventional dynamics is recovered.

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References

[1] Halperin B I and Hohenberg P C 1969 *Phys. Rev.* **177** 952
[2] Henley C L 1985 *Phys. Rev. Lett.* **54** 2030
[3] Kutasov D, Domany E and Pytte E 1987 *Phys. Rev.* B **35** 3354
[4] Bell S C and Southern B W 1988 *Phys. Rev.* B **38** 333
[5] Droz M, Kamphorst Leal Da Silva J and Malaspinas A 1986 *Phys. Lett.* **115A** 448
[6] Cornell S, Droz M and Menyhard N 1991 *J. Phys. A: Math. Gen.* **24** L201
[7] Glauber R J 1963 *J. Math. Phys.* **4** 294
[8] Kawasaki K 1966 *Phys. Rev.* **145** 224
[9] Hohenberg P C and Halperin B I 1977 *Rev. Mod. Phys.* **49** 435
Halperin B I, Hohenberg P C and Ma S K 1972 *Phys. Rev. Lett.* **29** 1548
Halperin B I, Hohenberg P C and Ma S K 1974 *Phys. Rev.* B **10** 139
[10] Deker U and Haake F 1979 *Z. Phys.* B **35** 281
[11] Kimball J C 1979 *J. Stat. Phys.* **21** 289
[12] Achiam Y 1980 *J. Phys. A: Math. Gen.* **13** L93
[13] Haake F and Thol K 1980 *Z. Phys.* B **40** 219
[14] Southern B W and Achiam Y 1992 in preparation
[15] Masui S, Southern B W and Jacobs A E 1989 *Phys. Rev.* B **39** 6925